

# $AdS_5$ black hole at $\mathcal{N}=2$ supergravity

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## Abstract

In this paper we consider three-charge none-extremal black hole at the five dimensional  $\mathcal{N}=2$  supergravity. We study thermodynamics of the  $AdS_5$  black hole with three equal charges ( $q_1 = q_2 = q_3 = q$ ). We obtain Schrödinger like equation and discuss the effective potential. Then we consider the case of the perturbed dilaton field background and find that the odd coefficients of the wave function to be appear. Also we find that the higher derivative corrections have not any effect on the first and second even coefficients of the wave function.

**Keywords:** AdS/CFT Correspondence;  $\mathcal{N} = 2$  Supergravity; Black holes; Dilaton Field; Higher Derivative Correction.

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# 1 Introduction

After 1997 the AdS/CFT correspondence has been introduced as a strong mathematical tools to study the gauge theories at the strong coupling [1-6]. The AdS/CFT correspondence relates a classical gravity on the AdS space to a gauge theory, with the conformal invariance, on the boundary of space-time. The main and known example of the AdS/CFT correspondence is the relation between the type IIB string theory in  $AdS_5 \times S^5$  space and  $\mathcal{N} = 4$  super Yang-Mills gauge theory on the 4-dimensional boundary of  $AdS_5$  space. Recent studies in the case of reduction supersymmetry have shown that there is the relation between the string theory and the gauge theory with the less supersymmetry [7-14].

So, in this paper we are going to consider the  $AdS_5$  black hole at the  $\mathcal{N} = 2$  supergravity background and discuss the massless scalar field in the black hole. The  $\mathcal{N} = 2$  supergravity theory is dual to the  $\mathcal{N} = 4$  SYM theory with finite chemical potential. Recently it is found that  $\mathcal{N} = 2$  supergravity is an ideal laboratory [14-18], also discussion of the AdS/CFT correspondence with the  $\mathcal{N} = 2$  supergravity background have been studied [7-19]. The procedure of finding quasi-normal modes of the massless scalar field in the black hole performed by calculation of the retarded Green's function. It is known that the imaginary part of the Green's function is related to the quasi-normal modes. In order to obtain quasi-normal modes at various backgrounds there are many methods which studied for example in the Refs. [20-38]. Here, for the first time we consider the problem of the charged non-extremal black hole at the  $\mathcal{N} = 2$  supergravity background. In the recent work [38] the quasi-normal modes of the massless scalar field in the AdS black hole and scalar glueballs in a holographic AdS/QCD model at the finite temperature considered. This model known as the soft-wall model [39, 40, 41]. In the Ref. [29] scalar glueballs spectrum at the finite temperature plasma has been studied by using AdS/QCD correspondence. Also the vector meson spectrum [39],  $m^2 = 4c(n+1)$ , scalar glueballs spectrum,  $m^2 = 4c(n+2)$ , and vector glueballs spectrum,  $m^2 = 4c(n+3)$ , have been discussed [40], where  $n = 0, 1, 2, \dots$  is the radial quantum number and the parameter  $\sqrt{c}$  plays the role of a mass scale. The glueballs are described by the massless scalar field in the black hole. According to the Maldacena dictionary spectrum of the scalar glueballs on the boundary of space-time are corresponding to the quasi-normal modes of the massless scalar fields in the black hole. In this formulation we deal with a five dimensional AdS space and a background of dilaton field  $\Phi(r)$ . The  $AdS_5$  line element in the Poincare coordinates is given by,

$$ds_{AdS}^2 = \frac{r^2}{L^2} \left[ -dt^2 + \sum_{i=1}^3 (dx^i)^2 + dr^2 \right], \quad (1)$$

where  $L$  denotes curvature radius of AdS space. In presence of background field  $\Phi(r)$ , into the bulk, one needs to rewrite the action as the following,

$$S = -\frac{N_c^2}{4\pi^2} \int d^5x \sqrt{-g} e^{-\Phi} \mathcal{L}, \quad (2)$$

where the factor  $e^{-\Phi} \mathcal{L}$  denotes the interaction of the dilaton field with some matter. So, in absence of some matter the lagrangian  $\mathcal{L}$  don't couple to the dilaton field. In this paper

we will consider the case of without matter. The context of this paper summarized as the following.

In the next section we consider the  $AdS_5$  black hole at the  $\mathcal{N} = 2$  supergravity theory in presence of the dilaton field background and try to obtain the black hole density of the specific heat. Then we extract conditions of stable/unstable transition of the theory which is corresponding to the confinement/deconfinement phase transition [42-45]. In the section 3 we obtain the massless scalar field equation of motion and rewrite it in the form of the Schrödinger equation. In order to find corresponding wave function we solve the Schrödinger like equation. Also we discuss the effective potential in the Schrödinger like equation. Then, by using the relation of retarded Green's function with the quasi-normal modes, in the section 4 we try to write the equations of the quasi-normal mode frequencies for the massless scalar field in the black hole. These equations may be solved numerically. In the section 5 we consider the case of the perturbed dilaton field background and find modified effective potential and wave functions. Finally, in the section 6, we consider higher derivative corrections of the  $AdS_5$  black hole at the  $\mathcal{N} = 2$  supergravity, and obtain the effect of the higher derivative terms on the effective potential. In the section 7 we summarized our results.

## 2 $\mathcal{N} = 2$ supergravity

We consider a soft-wall model with the dilaton field background  $\Phi(r) = cr^2$  and the non-extremal black hole of  $\mathcal{N} = 2$   $AdS_5$  supergravity,

$$\begin{aligned} ds^2 &= -\frac{f}{H^2}dt^2 + H(r^2 d\Omega_3^2 + \frac{dr^2}{f}), \\ f &= 1 - \frac{\eta}{r^2} + g^2 r^2 H^3, \\ H &= 1 + \frac{q}{r^2}, \end{aligned} \tag{3}$$

where  $g$  is the coupling constant and relates to the cosmological constant via  $\Lambda = -6g^2$ .  $q$  is called the black hole charge and it can be related to the non-extremality parameter  $\eta$  by using the relation  $q = \eta \sinh^2 \beta$  [11, 12, 13], where the  $\beta$  parameter is related to the electrical charge of the black hole. Notice that, in  $q \rightarrow 0$  ( $\eta \rightarrow 0$ ) limit, the line element in (3) is reduced to the line element of an extremal black hole (near extremal) with zero charge (infinitesimal charge). The coordinate  $r$  is axis along the black hole which defined in the interval  $0 \leq r \leq \infty$ . Let assume that the horizon of black hole is located at  $r = r_h$ . Also the boundary of space-time located at  $r \rightarrow \infty$ . There is the well known relation between the Hawking temperature, horizon radius and charge of black hole as the following [46],

$$T_H = \frac{2 + 3k - k^3}{2(1 + k)^{\frac{3}{2}}} \frac{r_h}{\pi L^2}, \tag{4}$$

where  $k \equiv \frac{q}{r_h^2}$  and  $L$  denotes curvature radius of AdS space which relates to the coupling constant via  $g = \frac{1}{L}$ . Clearly, the  $q \rightarrow 0$  ( $\eta \rightarrow 0$ ) limit of the equation (4) reduces to the

Hawking temperature of  $\mathcal{N} = 4$  SYM theory in 4 dimensions,  $T = \frac{r_h}{\pi L^2}$  [38]. Also one can see that the zero temperature obtained by taking  $r_h^2 = \frac{q}{2}$  in the equation (4). According to the Maldacena dictionary the temperature  $T$  is corresponding to the temperature of the field theory on the boundary. The  $\mathcal{N} = 2$   $AdS_5$  supergravity solution (3) is dual to the  $\mathcal{N} = 4$  SYM with finite chemical potential in Minkowski space. It can be shown by the following rescaling [46],

$$r \rightarrow \lambda^{\frac{1}{4}} r, \quad t \rightarrow \frac{t}{\lambda^{\frac{1}{4}}}, \quad \eta \rightarrow \lambda \eta, \quad q \rightarrow \lambda^{\frac{1}{2}} q, \quad (5)$$

and taking  $\lambda \rightarrow \infty$  limit while,

$$d\Omega_3^2 \rightarrow \frac{1}{L^2 \lambda^{\frac{1}{2}}} (dx^2 + dy^2 + dz^2), \quad (6)$$

and also we set  $r_0^4 \equiv \eta R^2$ . Then solution (3) reduces to the following,

$$\begin{aligned} ds^2 &= e^{2A(r)} \left[ -\frac{f}{H^2} dt^2 + H d\vec{X}^2 + \frac{H}{f} dr^2 \right], \\ f &= H^3 - \frac{r_0^4}{r^4}, \\ H &= 1 + \frac{q}{r^2}, \end{aligned} \quad (7)$$

where the geometric function  $A(r)$  defined as  $A(r) \equiv \ln \frac{r}{L}$ . The coordinate  $r$  defined in the interval  $r_0 \leq r \leq \infty$ , so  $r = \infty$  corresponds to the space boundary. It is interesting to consider special case of one charge black hole ( $q_1 = q, q_2 = q_3 = 0$ ). In that case the Hawking temperature reads as,

$$T_H = \frac{q + 2r_0^2}{2\pi L^2 \sqrt{q + r_0^2}}. \quad (8)$$

The radius of the horizon is given by,

$$r_h^2 = \frac{1}{2} \left( \sqrt{4r_0^4 + q^2} - q \right). \quad (9)$$

Also one can write the chemical potential in terms of the black hole charge and horizon radius,

$$\mu = \frac{r_h}{L^2} \sqrt{\frac{2q}{q + r_h^2}}. \quad (10)$$

Therefore the  $q = 0$  limit is equal to the zero chemical potential limit. Approximately, one can say that the  $\sqrt{-g}$  in the action (2) is the same for both  $q \neq 0$  and  $q = 0$  cases. The case of  $q = 0$  studied in the Ref. [38], but for the charged black hole one can obtain,

$$r_0^2 = \frac{\pi^2 T_H^2 L^4 - q}{2} \left[ 1 + \sqrt{1 + q \frac{4\pi^2 T_H^2 L^4 - q}{(\pi^2 T_H^2 L^2 - q)^2}} \right]. \quad (11)$$

Then by using the metrics (1) and (7) in the action (2) the  $S_{BH}$  obtained in terms of the  $r_0$ . The specific heat is important parameter to find the phase transition which will be calculated by the relation  $\mathcal{C}_{BH} \equiv -\beta^2 \frac{\partial^2}{\partial \beta^2} S_{BH}$ . In that case one can obtain,

$$\mathcal{C}_{BH} \propto \frac{\tilde{T}^2}{\sqrt{\tilde{T}^2 - q}} \left[ (6c - 9q + \frac{2c\tilde{T}^2}{\tilde{T}^2 - q} + 12\tilde{T}^2) e^{-\frac{c}{\tilde{T}^2 - q}} - 6\tilde{T}^2 \right] + \mathcal{O}(q^2), \quad (12)$$

where we defined  $\tilde{T} = \pi L^2 T_H$  and used  $\beta = \frac{1}{T_H}$ . This is clear that the case of  $q = 0$  recovers results of the Ref. [38]. In the  $c \rightarrow 0$  limit the sign of the specific heat is positive for  $q = 0$ , but in our case with  $q \neq 0$  the sign of the specific heat is depends to the black hole charge, so if  $\tilde{T}^2 > 1.5q$  then the charged black hole is in stable phase. In the Ref. [38] it is found that the specific heat changes the sign at  $\tilde{T}^2 \simeq 0.75$  (for  $c \neq 0$  and  $q = 0$ ). In presence of the dilaton field ( $c \neq 0$ ) and in unit of  $c$  one can find that the phase transition temperature from unstable to stable black hole increases for the case of charged black hole. For example, in the case of  $q = 1$  we find unstable/stable phase transition happen at  $\tilde{T}^2 \simeq 2.4$ , so the charged black hole is in stable phase for  $\tilde{T}^2 > 2.4$ .

### 3 Equation of motion

As we know from Maldacena dictionary, dual of the scalar glueball on the boundary of space-time is the massless scalar field in the bulk (black hole). In this section we want to solve the equation of motion for scalar field in the black hole and obtain wave functions. In order to obtain wave functions we try to rewrite the equation of motion in the form of Schrödinger equation. Then we find solutions of equation of motion by using asymptotic behavior of the Schrödinger like equation. The massless scalar field described by the following action,

$$S = -\frac{\pi^3 L^5}{4\kappa_{10}^2} \int d^5x \sqrt{-g} e^{-\Phi} g^{MN} \partial_M \phi \partial_N \phi, \quad (13)$$

where  $\Phi(r) = cr^2$  is dilaton field and  $g_{MN}$  is the metric given by the relation (7). The parameters  $L$  and  $\kappa_{10}$  are the radius of the AdS space and the ten-dimensional gravity constant respectively. Indices  $M$  and  $N$  run from 0 to 4, so coordinates  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) describe four-dimensional boundary and  $x^4 = r$  is extra coordinate along the black hole. By using the equation of motion for the scalar field  $\phi$  in the action (13) one can obtain,

$$\frac{e^\Phi}{\sqrt{-g}} \partial_r (\sqrt{-g} e^{-\Phi} g^{rr} \partial_r \phi) + g^{\mu\nu} \partial_\mu \partial_\nu \phi = 0. \quad (14)$$

Now, by using the following Fourier transformation,

$$\phi(r, x) = \int \frac{d^4k}{(2\pi)^4} e^{ik \cdot x} \bar{\phi}(r, k), \quad (15)$$

where  $k_\mu = (-\omega, p_i)$ , in the relation (14) one can write,

$$f(r)e^D \partial_r (e^{-D} f(r) \partial_r \bar{\phi}) + (H^3 \omega^2 - p^2 f(r)) \bar{\phi} = 0, \quad (16)$$

where  $p^2 = \Sigma_{i=1}^3 p_i^2$  and  $D \equiv cr^2 - 3A(r)$ . Now we introduce new coordinates  $\partial_{r_*} = -f(r) \partial_r$ , which is well known as Regge-Wheeler coordinates, so we can integrate it and find explicit expression of  $r_*$  in terms of  $r$ ,

$$r_* = r + \frac{1}{B-A} \left[ A^{\frac{3}{2}} \tan^{-1} \frac{r}{\sqrt{A}} - B^{\frac{3}{2}} \tan^{-1} \frac{r}{\sqrt{B}} \right], \quad (17)$$

where  $B \equiv \frac{1}{2}(q + \sqrt{q^2 + 4r_0^4})$  and  $A = -\frac{r_0^4}{B}$ . It is expected that the  $q = 0$  limit reduces to the Ref. [38], but we should note that our coordinates is different with that paper, so any differences at the  $q = 0$  limit is natural. Later we discuss how two results are coincide. Then we choose new variable as  $\psi = e^{-\frac{D}{2}} \bar{\phi}$ , where  $D \equiv cr^2 - 3 \ln \frac{r}{L}$ . By using above definitions in the equation (16) one can obtain following Schrödinger like equation,

$$\partial_{r_*}^2 \psi + \omega^2 \psi = V \psi, \quad (18)$$

where the effective potential defined as,

$$V = p^2 f(r) - \frac{D''}{2} + \frac{D'^2}{4} + \mathcal{O}(q), \quad (19)$$

where the prime denotes the derivative with respect to the  $r_*$  and  $\mathcal{O}(q) = -(\frac{3q}{r^2} + \frac{3q^2}{r^4} + \frac{q^3}{r^6})\omega^2$ . The explicit expression of the effective potential in terms of  $r$  written as the following relation,

$$V = \frac{f(r)}{r^2} \left[ p^2 r^2 + 2qc - \frac{3q}{r^2} - 4c \frac{r_0^4}{r^2} + 6 \frac{r_0^4}{r^4} + f(r) \left( c^2 r^4 + \frac{3}{4} - 3c - cr^2 \right) \right]. \quad (20)$$

In order to compare our results with the Ref. [38] we change our coordinates, so in the new coordinates we have  $f(z) = 1 + qz^2 - (\frac{z}{z_0})^4$  ( $z_0 \sim \frac{1}{r_0}$ ) and  $D = cz^2 + 3 \ln \frac{z}{L}$ , so the effective potential read as,

$$V = \frac{f(z)}{z^2} \left[ p^2 z^2 + \frac{15}{4} + \frac{9}{4} \left( \frac{z}{z_h} \right)^4 + 2cz^2 \left( 1 + \left( \frac{z}{z_h} \right)^4 \right) + c^2 z^4 f(z) + \frac{3}{4} q z^2 \right]. \quad (21)$$

In the Fig. 1 we plot the effective potential (21) in terms of the radial coordinate  $z$  for  $q = 1$ .

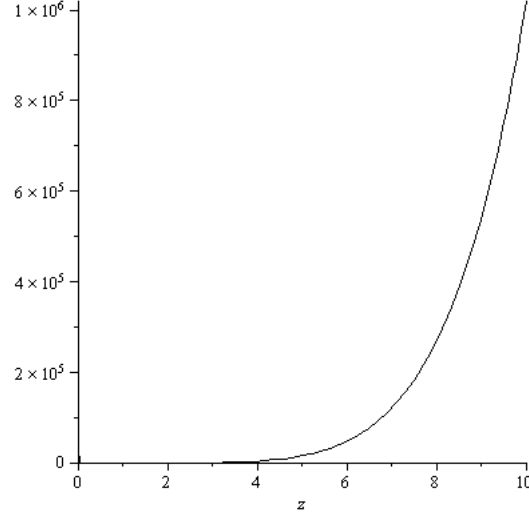


Figure 1: Plot of the effective potential in terms of  $z$  for  $q = c = p = 1$ .

Corresponding to the asymptotical behavior of the equation of motion of the scalar field one can write two solutions which satisfy the Schrödinger like equation,

$$\begin{aligned}\psi_1 &= z^{\frac{5}{2}}[1 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \dots] \\ \psi_2 &= z^{-\frac{3}{2}}[1 + b_1 z + b_2 z^2 + b_3 z^3 + \dots] + b_4 \psi_1 \ln(cz^2).\end{aligned}\quad (22)$$

Only none-zero coefficients are  $a_2$ ,  $a_4$ ,  $b_2$  and  $b_4$ . One can interpreted  $\psi_2 - b_4 \psi_1 \ln(cr^2)$  as a source of an operator on the boundary. In the other hand from the Ref. [38] we know that it is glueball operator which is dual of the massless scalar field in to the bulk. The none-zero coefficients of the wave function (22) are given by,

$$\begin{aligned}a_2 &= -\frac{\omega^2 - p^2 - 2c + 8q}{12}, \\ a_4 &= \frac{(\omega^2 - p^2 - 2c + 8q)^2}{384} + \frac{1}{2z_h^4} + \frac{c^2}{32} + \frac{q}{32}(p^2 + 2c - 8q), \\ b_2 &= \frac{\omega^2 - p^2 - 2c + 8q}{4}.\end{aligned}\quad (23)$$

These coefficients reduce to that in the Ref. [38] at the  $q = 0$ . We will use above solutions to construct retarded Green's function then one can extract quasi-normal modes.

One can see that at the  $z \rightarrow z_h$  ( $z_h^4 = \frac{z_0^4}{4}(qz_0^2 + \sqrt{4 + q^2 z_0^4})^2$ ) the effective potential vanishes. Therefore the Schrödinger like equation has the other solutions as incoming and outgoing wave functions which denoted by  $\psi_+$  and  $\psi_-$ . These solutions obtained by using this fact that the effective potential vanishes at the horizon and the Schrödinger like equation takes form of the free particle equation with solution of  $\exp(\pm i\omega r_*)$ . The negative sign interpreted as the incoming plane wave in to the horizon and the positive sign interpreted as the outgoing



plane wave from the horizon. Therefore one can write the following expansion as the near horizon solution of the Schrödinger like equation,

$$\psi_{\pm} = e^{\pm i\omega r_*} \left[ 1 + a_{1(\pm)} \left(1 - \frac{z}{z_h}\right) + a_{2(\pm)} \left(1 - \frac{z}{z_h}\right)^2 + \dots \right]. \quad (24)$$

The  $\psi_{\pm}$  form the basis of any other wave functions, also it may be expand  $\psi_{\pm}$  in terms of  $\psi_1$  and  $\psi_2$ . Therefore one can relate solutions in (22) to (24) and vise versa,

$$\begin{aligned} \psi_{\pm} &= \mathcal{A}_{(\pm)} \psi_2 + \mathcal{B}_{(\pm)} \psi_1, \\ \psi_{2,1} &= \mathcal{C}_{(2,1)} \psi_- + \mathcal{D}_{(2,1)} \psi_+, \end{aligned} \quad (25)$$

where  $\mathcal{A}_{(+)}$ ,  $\mathcal{A}_{(-)}$ ,  $\mathcal{B}_{(+)}$ ,  $\mathcal{B}_{(-)}$ ,  $\mathcal{C}_{(2)}$ ,  $\mathcal{C}_{(1)}$ ,  $\mathcal{D}_{(2)}$  and  $\mathcal{D}_{(1)}$  are  $\omega$  and  $p$  dependent coefficients which determined by using boundary condition and related to each other by the following relation,

$$1 = \begin{pmatrix} \mathcal{A}_{(-)} & \mathcal{B}_{(-)} \\ \mathcal{A}_{(+)} & \mathcal{B}_{(+)} \end{pmatrix} \begin{pmatrix} \mathcal{C}_{(2)} & \mathcal{D}_{(2)} \\ \mathcal{C}_{(1)} & \mathcal{D}_{(1)} \end{pmatrix}. \quad (26)$$

This relation will be useful to find quasi-normal modes.

Before end of this section we would like to discuss special behaviors of the effective potential. Here we set  $p = 0$ , and rewrite the effective potential for  $c = 0$  which is corresponding to the case of the charged black hole without the dilaton field. In that case the effective potential is given by,

$$V = \frac{f(z)}{z^2} \left[ \frac{15}{4} + \frac{9}{4} \left( \frac{z}{z_h} \right)^4 + \frac{3}{4} q z^2 \right]. \quad (27)$$

It is clear that the effective potential enhanced due to the black hole charge.

## 4 Imaginary part of the retarded Green's function

In order to obtain quasi-normal modes of scalar field  $\phi$  in the black hole we should find retarded Green's function. As we know the imaginary part of the retarded Green's function is known as spectral function. It yields us to frequency of the quasi-normal modes. We use results of the Ref. [38] to write the retarded Green's function. In that case by using equation of motion one can rewrite the action (13) as the following,

$$S = -\frac{\pi^3 L^5}{4\kappa_{10}^2} \int d^4 x dz \partial_z \left( \frac{f}{H} e^{-2A(z)} \sqrt{-g} e^{-\Phi} \phi \partial_z \phi \right), \quad (28)$$

where  $\phi_k(z)$  satisfies equation of motion and one can expand it in terms of obtained solution in the previous section. So, for the case of finite temperature one can write,

$$\phi_k(z) = e^{\frac{D}{2}} \left[ \psi_2(z) + \frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}} \psi_1(z) \right]. \quad (29)$$

In the action (28) we should integrate over  $z$  and set  $\phi(z_h) = 0$ . Then after some calculations it is found that the imaginary part of the retarded Green's function is proportional to the  $\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}}$ . This quantity called spectral function. In the AdS/CFT point of view the spectral function is in the AdS side which is dual of the correlation function in the CFT side. Therefore we use the following relation [38] to obtain imaginary part of the retarded Green's function,

$$\frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}} = \frac{\partial_z \psi_- \psi_2 - \psi_- \partial_z \psi_2}{\partial_z \psi_- \psi_1 - \psi_- \partial_z \psi_1}. \quad (30)$$

In the above expression we should evaluate functions in the near horizon limit. In that case one can obtain,

$$Im \frac{\mathcal{B}_{(-)}}{\mathcal{A}_{(-)}} = \omega \frac{\psi_2 \partial_z \psi_1 - \psi_1 \partial_z \psi_2}{(\omega \psi_1)^2 + (\partial_z \psi_1)^2}, \quad (31)$$

where  $\psi_1$  and  $\psi_2$  are given by the equation (22) at  $z \rightarrow z_h$  limit. Then one can find the value of the expression (31) numerically. In that case we draw the zero-momentum case of the imaginary part of the retarded Green's function (spectral functions) in terms of frequency for various values of temperature in the Fig. 2. Note that as the temperature increases the width of peaks in the spectral function decreases. Accordingly, the lifetimes of glueball quasiparticle states increase with the temperature. This situation completely different with the Ref. [38]. We should note that the range of temperatures where we found glueball excitations, the theory is in an unstable phase. However we can continue increasing the temperature to stable phase of theory but glueball excitations (peaks of the spectral function) disappear.

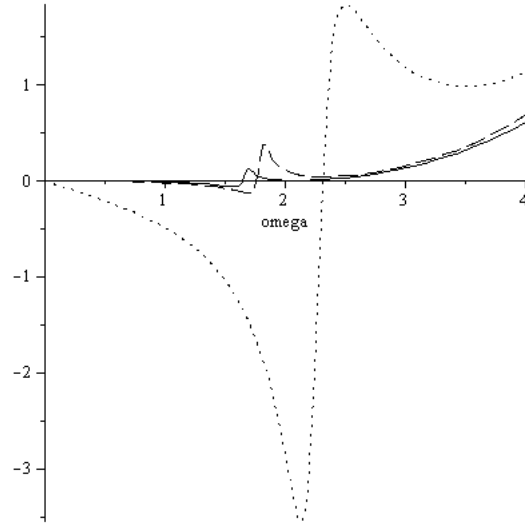


Figure 2: Spectral functions for  $p = 0$ ,  $c = q = 1$  and selected values of temperature. Note that the first peak of the spectral functions find around  $\omega = 1.6$ . Dotted, dashed and solid lines drawn for  $\tilde{T}^2 = 0.4, 1.6$  and  $2.4$  respectively. It is interesting that the first peak is corresponding to phase transition temperature.

In the Fig. 3 we draw graph of spectral function in terms of  $\omega$  for fixed value of  $p = 0$ ,  $c = 1$ ,  $\tilde{T}^2 = 0.4$  and various values of black hole charge. It show that the peaks of spectral function placed around  $\omega = 2.5$ . It is corresponding to dotted line in the Fig. 2. According to the Fig. 3 the peaks of spectral function appears at  $q > 0.8$  so, for the  $q < 0.8$  glueball excitations disappear. On the other hand by increasing the black hole charge the life time of glueball quasiparticle states increases.

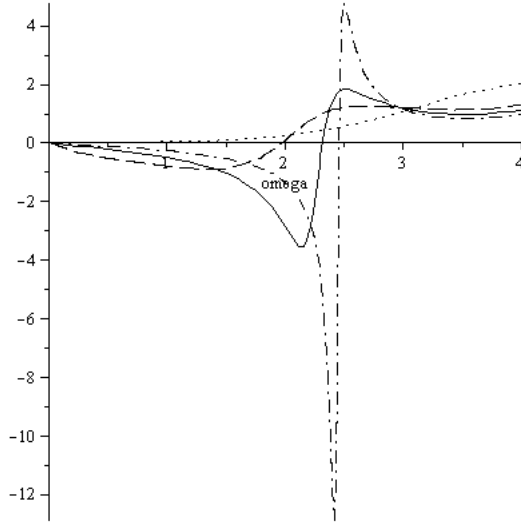


Figure 3: Spectral functions for  $p = 0$ ,  $c = 1$ ,  $\tilde{T}^2 = 0.4$  and selected values of the black hole charge. Note that the peaks of the spectral functions find around  $\omega = 2.5$ . Dotted, dashed, solid and dash dotted lines drawn for  $q = 0, 0.8, 1$  and  $1.2$  respectively.

There are many different ways to obtain the quasi-normal modes of the scalar field in the black holes. One of them is the power series method [47] which is suitable for asymptotically AdS space-time. In that case we begin with the Schrödinger like equation (18). One can introduce new function such as  $\psi = \varphi e^{-i\omega r_*}$  and put in the equation (18), then one can obtain,

$$\partial_{r_*}^2 \varphi - 2i\omega \partial_{r_*} \varphi = V \varphi. \quad (32)$$

Now, by using the relation  $\partial_{r_*} = -f(z)\partial_z$  we yield the following equation,

$$f(z) \frac{\partial^2 \varphi}{\partial z^2} + \left( 2i\omega + 2qz - \frac{4z^3}{z_h^4} \right) \frac{\partial \varphi}{\partial z} - \frac{V(z)}{f(z)} \varphi = 0. \quad (33)$$

Because of a regular singularity at  $z = z_h$  one can write a possible solution of the equation

(33) as,

$$\varphi_-(r) = \sum_m a_{m(-)} \left(1 - \frac{z}{z_h}\right)^m. \quad (34)$$

In order to find coefficients  $a_{m(-)}$  we should put  $\varphi_-$  into the equation (33), then by using the Dirichlet boundary condition we have,

$$\sum_m a_{m(-)} = 0. \quad (35)$$

This leads us to obtain the roots of the equation (35). This procedure can be performed numerically, then the quasi-normal frequencies obtained.

## 5 Perturbed dilaton field background

The light scalar and vector glueballs spectra in a holographic QCD with a perturbed dilaton field background originally studied in the Ref. [40], also AdS/QCD duality in a perturbed AdS-dilaton background was considered [41]. In the mentioned Refs. the equation of motion for the bulk-to-boundary propagator and AdS two-point correlation function obtained and concluded that the AdS dynamics not affected by dilaton perturbations. Now we would like to apply perturbed dilaton background to the Schrödinger like equation and obtain effect of the perturbation on the effective potential and solutions of the wave function.

It is known that the dilaton field background and geometric function should be satisfy the Regge behavior. The simplest choice consistent with these conditions given in the previous sections. Also there is another choice for the dilaton field background  $\Phi(z)$  which obey the Regge behavior. One can perturbed dilaton field background as the following [40],

$$\begin{aligned} \Phi(z) &= cz^2 + \sqrt{c}\lambda z \\ A(z) &= -\ln \frac{z}{L}, \end{aligned} \quad (36)$$

where  $\lambda$  is the perturbation parameter which is small dimensionless parameter. The choice (36) does not effectively modifies the Regge behavior of the spectrum. It is found that for small values of the parameter  $\lambda$  the first three spectra of the scalar glueball modified as,

$$\begin{aligned} m_0^2 &= 8c + \lambda \frac{3\sqrt{\pi}}{2}, \\ m_1^2 &= 12c + \lambda \frac{27\sqrt{\pi}}{16}, \\ m_2^2 &= 16c + \lambda \frac{237\sqrt{\pi}}{128}. \end{aligned} \quad (37)$$

Also the first three spectra of the vector glueball modified as,

$$\begin{aligned} m_0^2 &= 12c + \lambda \frac{189\sqrt{\pi}}{128}, \\ m_1^2 &= 16c + \lambda \frac{105\sqrt{\pi}}{64}, \\ m_2^2 &= 20c + \lambda \frac{14667\sqrt{\pi}}{8192}. \end{aligned} \quad (38)$$

It have shown that the mass splitting between vector and scalar qlueballs increases if  $\lambda$  be negative. These spectra, for the case of  $q = 0$  obtained in the Ref. [40], now we would like to obtain the modified waves functions and the effective potential corresponding to the Schrödinger like equation.

By using the perturbed dilaton background field (36) also one can obtain Schrödinger like equation (18) where the effective potential modified as the following,

$$\begin{aligned} V &= \frac{f(z)}{z^2} \left[ p^2 z^2 + \frac{15}{4} + \frac{9}{4} \frac{z^4}{z_h^4} + 2cz^2 \left( 1 + \frac{z^4}{z_h^4} \right) + c^2 z^4 f(z) + \frac{3}{4} q z^2 \right] \\ &+ \frac{f(z)}{z^2} \sqrt{c} \lambda \left[ 2 \frac{z^5}{z_h^5} - q z^3 + \left( \frac{\sqrt{c} \lambda z^2}{4} + c z^3 + \frac{3}{2} z \right) f(z) \right]. \end{aligned} \quad (39)$$

In the Fig. 4 we plot the effective potential (39) in terms of the radial coordinate  $z$  for  $q = \lambda = 1$ . We find that the dilaton perturbation increases the value of the effective potential.

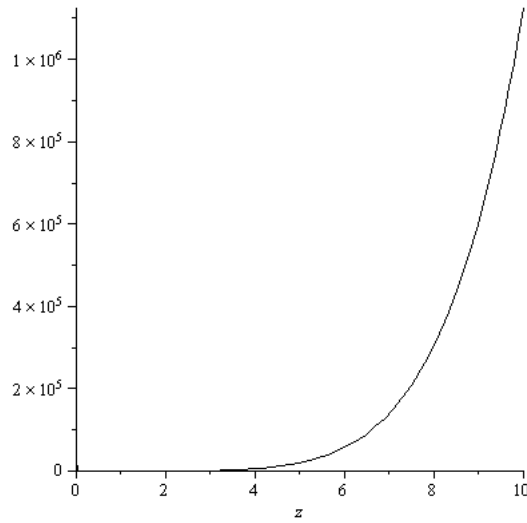


Figure 4: Plot of the effective potential in terms of  $z$  for  $q = \lambda = c = p = 1$ .

Also the coefficients of the solution (22) modified as the following,

$$\begin{aligned}
a_1 &= \frac{9\sqrt{c}\lambda}{8}, \\
a_2 &= -\frac{\omega^2 - p^2 - 2c + 8q}{12} + \frac{31}{192}c\lambda^2, \\
a_3 &= \frac{\sqrt{c}\lambda}{42} \left[ \frac{67}{64}\sqrt{c}\lambda^2 + \frac{5}{2}p^2 + 7c - \frac{5}{2}\omega^2 - \frac{253}{4}q \right], \\
a_4 &= \frac{(q^2 + 2c - \omega^2 - 8q)^2}{384} + \frac{1}{2z_h^4} + \frac{c^2}{32} + q(p^2 + 2c - 8q), \\
&\quad + c\lambda^2 \left[ \frac{11}{8}c + \frac{15}{168}(p^2 - \omega^2) - \frac{55}{112}q + \frac{35}{192}(p^2 + 2c - \omega^2 - 8q) \right] + \mathcal{O}(\lambda^4), \\
b_1 &= -\frac{6\sqrt{c}\lambda}{17}, \\
b_2 &= \frac{\omega^2 - p^2 - 2c + 8q}{4} - \frac{3c\lambda^2}{4}, \\
b_3 &= \frac{2\sqrt{c}\lambda}{51} \left[ 15 + \frac{99}{4}(\omega^2 - p^2 - 2c + 8q) \right] + \mathcal{O}(\lambda^3). \tag{40}
\end{aligned}$$

The main consequence of perturbed dilaton background is appearance of the odd coefficients ( $a_1$ ,  $a_3$ ,  $b_1$  and  $b_3$ ).

Now, we would like to discuss about special behaviors of the modified effective potential. At the zero-temperature limit, where  $z_h \rightarrow \infty$  and  $q = 0$ , the effective potential takes the following form,

$$V_{T=0}(z) = \frac{1}{z^2} \left[ (p^2 + 2c)z^2 + \frac{15}{4} + c^2z^4 + \frac{c\lambda^2z^2}{4} + \sqrt{c}\lambda z \left( \frac{3}{2} + cz^2 \right) \right]. \tag{41}$$

Also at the  $c \rightarrow 0$  limit for finite temperature one can obtain,

$$V_{c=0}(z) = \frac{f(z)}{z^2} \left( p^2z^2 + \frac{15}{4} + \frac{9}{4}\frac{z^4}{z_h^4} + \frac{3}{4}qz^2 \right), \tag{42}$$

which is corresponding to the case of without dilaton field. Therefore in the above expression the parameter  $\lambda$  is not exist.

By increasing the temperature the effective potential changes. It is known that (for non-perturbed dilaton field background) at high-temperature there are no bound states. This situation is similar to the recent case with perturbed dilaton field background.

At low-temperature limit and  $q = 0$  one can write the effective potential approximately as

the following,

$$\begin{aligned}
V(z) &= \frac{1}{z^2} \left[ \frac{15}{4} - \frac{3}{2} \frac{z^4}{z_h^4} + 2cz^2 + c^2 z^4 - 2c^2 \frac{z^8}{z_h^4} \right] \\
&+ \frac{1}{z^2} \left[ \sqrt{c}\lambda \left( \frac{3}{2}z + cz^3 - \frac{3}{2} \frac{z^4}{z_h^4} + \frac{1}{2} \frac{z^5}{z_h^4} - 2c \frac{z^7}{z_h^4} \right) \right] \\
&+ \frac{1}{z^2} \left[ c\lambda^2 \left( \frac{z^2}{4} - \frac{z^6}{2z_h^4} \right) \right].
\end{aligned} \tag{43}$$

Then by using the relation  $z_h = \frac{1}{\pi T}$  one can obtain,

$$\begin{aligned}
V(z) &= \frac{1}{z^2} \left[ \frac{15}{4} + 2cz^2 + c^2 z^4 + \sqrt{c}\lambda z \left( \frac{3}{2} + cz^2 \right) + \frac{c\lambda^2 z^2}{4} \right] \\
&- z^2 \pi^4 T^4 \left[ \frac{3}{2} + 2c^2 z^4 - \sqrt{c}\lambda z \left( \frac{1}{2} - \frac{3}{2z} - 2cz^2 \right) + \frac{c\lambda^2 z^2}{2} \right].
\end{aligned} \tag{44}$$

The modified effective potential at low-temperature has a constant term as  $(2 + \frac{\lambda^2}{4})c$ , a perturbed oscillator like potential term as  $c^2 z^2 + c^{\frac{3}{2}} \lambda z$ , a term with the bare infinity at  $z = 0$  as  $\frac{15}{4z^2} + \frac{3\sqrt{c}\lambda}{2z}$ , and finally temperature corrections terms. The expression (44) is agree with the relation (43) at  $T = 0$  limit.

## 6 Higher derivative corrections

In this section we would like to calculate the effect of higher derivative terms on the effective potential and wave functions for the case of none-perturbed dilaton field. In this way we use results of the Ref. [48], where the first-order correction of the solution (3) is given by,

$$\begin{aligned}
f &= 1 + \frac{r^2 + q}{L^2} - \frac{\eta}{r^2} - \frac{1}{r^4} \left( \frac{\alpha}{r^2 + q} + \beta \right), \\
H &= 1 + \frac{q}{r^2} - \frac{\beta L^2}{5r^4(r^2 + q)},
\end{aligned} \tag{45}$$

where we define,

$$\begin{aligned}
\alpha &= c_1 \left( \frac{q(q + \eta)}{72L^2} - \frac{\eta^2}{96} \right), \\
\beta &= \frac{c_1 q(q + \eta)}{9L^2},
\end{aligned} \tag{46}$$

and  $c_1$  is arbitrary constant. In this background the horizon radius  $r_h$  is the root of the following equation,

$$1 + \frac{r^2 + q}{L^2} - \frac{\eta}{r^2} - \frac{1}{r^4} \left( \frac{\alpha}{r^2 + q} + \beta \right) = 0. \tag{47}$$

Again we use rescaling (5) and take  $\lambda \rightarrow \infty$  limit, so we get,

$$\begin{aligned} f &= 1 + \frac{q}{r^2} - \frac{r_0^4}{r^4} + \frac{c_1 r_0^8}{96 L^2 r^6 (r^2 + q)}, \\ H &= 1 + \frac{q}{r^2} - \frac{c_1 r_0^4}{9 L^4 r^4 (r^2 + q)}, \end{aligned} \quad (48)$$

and now the radius  $r_h$  is the root of the following relation,

$$r^8 + 2qr^6 + (q^2 - r_0^4)r^4 - qr_0^4r^2 + \frac{c_1 r_0^4}{L^2} = 0. \quad (49)$$

For the special case of  $q = 0$  one can obtain solution of the above relation as the following,

$$r_h^4 = \frac{r_0^4}{2} \left( 1 + \sqrt{1 - \frac{4c_1}{L^2}} \right). \quad (50)$$

Then similar to the section 3 we change our coordinates so we have,

$$\begin{aligned} f &= 1 - \frac{z^4}{z_0^4} + \frac{c_1 z^8}{96 L^2 z_0^8}, \\ H &= 1 - \frac{c_1 z^6}{9 L^4 z_0^4}. \end{aligned} \quad (51)$$

In this case the effective potential modified as the following,

$$V = \frac{f(z)}{z^2} \left[ p^2 z^2 + \frac{15}{4} + \frac{9}{4} \left( \frac{z}{z_h} \right)^4 + 2c z^2 \left( 1 + \frac{z^4}{z_h^4} - \frac{c_1}{32 L^2} \frac{z^8}{z_h^8} \right) + c^2 z^4 f(z) + \frac{11 c_1}{128 L^2} \frac{z^8}{z_h^8} \right], \quad (52)$$

where  $f(z)$  is given by the equation (51) and  $z_h^4 = \frac{4Lz_0^4}{c_1} (12L + \sqrt{144L^2 - 6c_1})$ . Comparing the equations (52) and (21) tell us that the higher order correction terms obtained as,

$$- \frac{c_1}{32 L^2} \frac{z^8}{z_h^8} f(z) \left( C + \frac{11}{4z^2} \right), \quad (53)$$

which decreases the effective potential if  $z^8 < \frac{z_h^8}{1 - \frac{c_1}{96 L^2}}$  and increases the effective potential if  $z^8 > \frac{z_h^8}{1 - \frac{c_1}{96 L^2}}$ , so at the horizon the higher order corrections have no effect on the effective potential, actually in the case of higher order corrections the effective potential vanishes at the horizon just like all previous cases. Finally we can investigate the effect of the higher order corrections on the wave function (22). In that case it is clear that the higher order terms have no effect on the first and second even coefficients, so by setting  $q = 0$  in the relation (23) we give same coefficients. The lowest coefficient which involves higher derivative corrections is  $a_6$ . In the Fig. 5 we plot simultaneously the effective potential for the cases of perturbed dilaton, higher derivative and without them for  $q = 0$ .



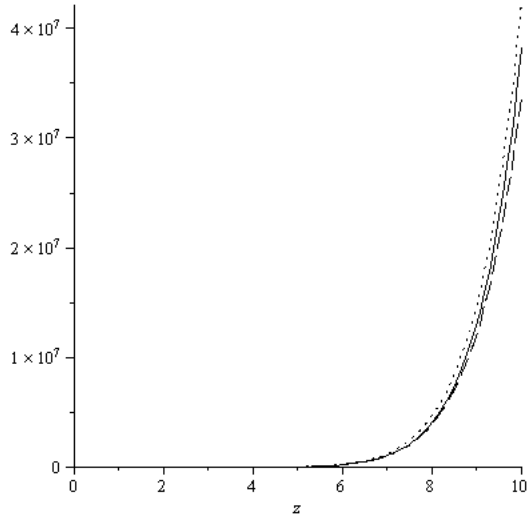


Figure 5: Plot of the effective potential in terms of  $z$  for  $q = \lambda = c_1 = 0$  (solid line),  $q = c_1 = 0, \lambda = 1$  (dotted line), and  $q = \lambda = 0, c_1 = 1$  (dashed line).

## 7 Conclusion

First of all we reviewed the  $\mathcal{N} = 2$   $AdS_5$  supergravity which includes a none-extremal black hole with three equal charges. We know that the  $\mathcal{N} = 2$   $AdS_5$  supergravity solution is dual to the  $\mathcal{N} = 4$  SYM with finite chemical potential. We used this duality and calculated the specific heat and found that the unstable/stable phase transition happen at  $\tilde{T}^2 \simeq 2.4$ . Comparing this result with the case of the zero charge [38] tell us that the effect of black hole charge is increasing the phase transition temperature. Then we obtained wave functions and the effective potential and found dependence of the coefficients and the effective potential on the black hole charge. Then we discussed about the imaginary part of the retarded Green's function numerically and found that by increasing the black hole charge the life time of glueball quasiparticle states increases.

The effect of the perturbed dilaton field background on the wave function and the effective potential studied in the section 5, we found that the odd coefficients of the wave function raised. We obtained the effective potential for the case of zero-temperature and zero dilaton field, also we obtained the case of the low-temperature and shown that the constant and oscillator like terms modified by square and linear form of the perturbation parameter respectively. In that case it is interesting to consider perturbed geometric function [40, 41]. Finally we considered the higher derivative correction which have no any contribution in the first and second even coefficients, so the first effect of the higher derivative terms is in the

$a_6$ . The effective potential modified by the two terms proportional to  $z^8 f(z)$  and  $z^6 f(z)$ . For the future work it is interesting to consider  $AdS_5$  black hole with three different charges. We conclude that the dilaton perturbation increases the effective potential, but higher derivative correction decreases the effective potential for  $z < z_h$ .

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